Quadratic APN Polynomials in Few Terms in Small Dimensions

Bo Sun

General Background
Cryptosystems and S-boxes
Vectorial Boolean Functions and Attacks
Nonliearity
Differential Uniformity

Equivalences between Vectorial Boolean Functions
Three Equivalences
Infinite Families of APN Functions
Classification of APN Functions for Small n

Experiment
Construction of Functions
Procedures
Results and Conclusions

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Bo Sun

University of Bergen, Norway

The 2nd International Workshop on Boolean Functions and their Applications

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Outline

1. General Background
   - Cryptosystems and S-boxes
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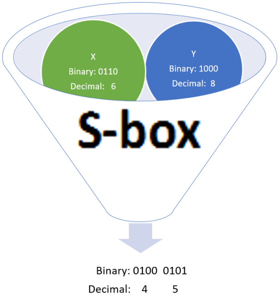
S-boxes

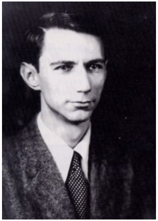
Any S-box substitutes m bits of value from one finite field to other n bits of value from the other finite field, and both sets' characteristic are 2.
S-boxes can be implemented as lookup tables.

Example of lookup table:

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16



Why S-boxes are Critical for Block Ciphers?



Claude E. Shannon (1916-2001)

- Two Properties that a good cryptosystem should have: Confusion and Diffusion
- S-boxes are important because:
  - They are the only nonlinear component in block cipher;
  - They provide confusion to symmetric block cipher;
  - There is strong connection between properties of S-boxes and resistance to many cryptographic attacks.

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Vectorial Boolean Functions and Attacks

For n and m positive integers

Boolean functions: $f : \mathbb{F}_2^n \to \mathbb{F}_2$

Vectorial Boolean functions: $F : \mathbb{F}_2^n \to \mathbb{F}_2^m$

- Linear attacks – Nonlinearity
- Differential attacks – Differential Uniformity
- ...

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Nonlinearity of Vectorial Boolean Functions

$F : \mathbb{F}_2^n \to \mathbb{F}_2^m$

Nonlinearity of Vectorial Boolean function: Minimum Hamming distance between all nonzero linear combinations of the boolean functions over $\mathbb{F}_2^n$ and component functions of F.

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Resistance to Linear Attacks

High nonlinearity $N(F)$ is necessary to resist linear attacks.

- Universal upper bound: $N(F) \leq 2^{n-1} - 2^{\frac{n}{2}-1}$;
- F is bent if $N(F) = 2^{n-1} - 2^{\frac{n}{2}-1}$;
- Bent functions are optimal against linear attacks;
- Bent functions exist iff: n is even and $m \leq n/2$;
- When $n = m$, n is odd, the upper bound is : $N(F) \leq 2^{n-1} - 2^{\frac{n-1}{2}}$;
- F is Almost Bent(AB) if $N(F) = 2^{n-1} - 2^{\frac{n-1}{2}}$, $n = m$ and n is odd;
- When n = m, n is even, it was conjectured that upper bound is: $N(F) \leq 2^{n-1} - 2^{\frac{n}{2}}$.

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Differential Uniformity of Vectorial Boolean Functions

$F : \mathbb{F}_2^n \to \mathbb{F}_2^m$ is differentially δ-uniform if the equations:

$$F(x+a) - F(x) = b, \qquad \forall a \in \mathbb{F}_2^n \backslash \{0\}, \quad \forall b \in \mathbb{F}_2^m,$$

have at most δ solutions.

Resistance to Differential Attacks

Low differential uniformity is necessary.

$F : \mathbb{F}_{2^n} \to \mathbb{F}_{2^m}$

- F is Perfect Nonlinear(PN) function if it is $2^{(n-m)}$-uniform;
- PN is optimal against differential attack;
  - F is bent iff it is PN;
  - PN is the highest nonlinearity and lowest uniformity
- When $n = m$, F is Almost Perfect Nonlinear(APN) function if it is 2-uniform;
- Every AB function is APN function. The converse is not true.

Optimal Functions on Uniformity and Linearity

Table 1. Optimal Functions on Uniformity and Linearity from $\mathbb{F}_{2^n}$ to $\mathbb{F}_{2^m}$

Conditions	**Functions' Name with Lowest Uniformity**	**Uniformity**	**Functions' Name with Highest Nonlinearity**	**Nonlinearity**
$m \leqslant n/2$	PN (or bent)	2^{n-m}	bent (or PN)	$2^{n-1} - 2^{\frac{n}{2}-1}$
$n/2 < m < n$	-	$> 2^{n-m}$	-	$\leqslant 2^{n-1} - \frac{1}{2}\left(3 \cdot 2^n - 2 - \frac{2(2^n-1)(2^{n-1}-1)}{(2^m-1)}\right)^{1/2}$
$m = n$, n is odd	APN	2	AB (or maximal nonlinear)	$2^{n-1} - 2^{\frac{n-1}{2}}$
$m = n$, n is even			maximal nonlinear	$\leqslant 2^{n-1} - 2^{\frac{n}{2}}$ (Conjectured as highest)

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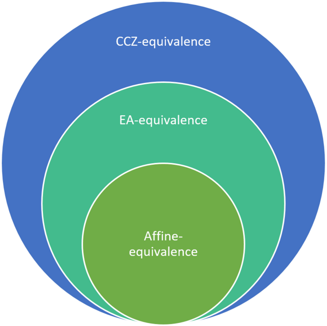
Three Kinds of Equivalences

Equivalent relation which has invariant differential uniformity and nonlinearity, APN-ness:

- Affine Equivalence
- Extended Affine Equivalence(EA-equivalence)
- Carlet-Charpin-Zinoviev equivalence(CCZ-equivalence)



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APN Power Functions

Table 2. Known families of APN power functions x^d on $\mathbb{F}_{2^n}$

Functions	Exponents d	Conditions
Gold	$2^i + 1$	$\gcd(i, n) = 1,\ 1 \leq i < n/2$
Kasami	$2^{2i} - 2^i + 1$	$\gcd(i, n) = 1,\ 2 \leq i < n/2$
Welch	$2^m + 3$	$n = 2m + 1$
Niho	$2^m + 2^{\frac{m}{2}} - 1$, m even $2^m + 2^{\frac{3m+1}{2}} - 1$, m odd	$n = 2m + 1$
Inverse	$2^{n-1} - 1$	$n = 2m + 1$
Dobbertin	$2^{4m} + 2^{3m} + 2^{2m} + 2^m - 1$	$n = 5m$

Conjecture: Up to CCZ-equivalence, the list is complete.

Quadratic APN Polynomials (I)

Table 3. Known families of quadratic APN polynomials CCZ-inequivalent to power functions on $\mathbb{F}_{2^n}$

N°	Functions	Conditions
1-2	$x^{2^s+1} + \alpha^{2^k-1} x^{2^{ik}+2^{mk+s}}$	$n = pk$, $\gcd(k,p) = \gcd(s,pk) = 1$, $p \in \{3,4\}$, $i = sk \bmod p$, $m = p - i$, $n \geq 12$, α primitive in $\mathbb{F}_{2^n}^*$
3	$x^{2^{2i}+2^i} + bx^{q+1} + cx^{q(2^{2i}+2^i)}$	$q = 2^m$, $n = 2m$, $\gcd(i,m) = 1$, $\gcd(2^i+1, q+1) \neq 1$, $cb^q + b \neq 0$, $c \notin \{\lambda^{(2^i+1)(q-1)}, \lambda \in \mathbb{F}_{2^n}\}$, $c^{q+1} = 1$
4	$x(x^{2^i} + x^q + cx^{2^i q})$ $+x^{2^i}(c^q x^q + sx^{2^i q}) + x^{(2^i+1)q}$	$q = 2^m$, $n = 2m$, $\gcd(i,m) = 1$, $c \in \mathbb{F}_{2^n}$, $s \in \mathbb{F}_{2^n} \setminus \mathbb{F}_q$, $X^{2^i+1} + cX^{2^i} + c^q X + 1$ is irreducible over $\mathbb{F}_{2^n}$

Quadratic APN Polynomials (II)

Table 3. Continued

N°	Functions	Conditions
5	$x^3 + a^{-1}\mathrm{tr}_1^n(a^3x^9)$	$a \neq 0$
6	$x^3 + a^{-1}\mathrm{tr}_3^n(a^3x^9 + a^6x^{18})$	$3 \mid n$, $a \neq 0$
7	$x^3 + a^{-1}\mathrm{tr}_3^n(a^6x^{18} + a^{12}x^{36})$	$3 \mid n$, $a \neq 0$
8-10	$ux^{2^s+1} + u^{2^k}x^{2^{-k}+2^{k+s}} +$ $vx^{2^{-k}+1} + wu^{2^k+1}x^{2^s+2^{k+s}}$	$n = 3k$, $\gcd(k,3) = \gcd(s,3k) = 1$, $v, w \in \mathbb{F}_{2^k}$, $vw \neq 1$, $3 \mid (k+s)$, u primitive in $\mathbb{F}_{2^n}^*$
11	$\alpha x^{2^s+1} + \alpha^{2^k}x^{2^{k+s}+2^k} +$ $\beta x^{2^k+1} + \sum_{i=1}^{k-1}\gamma_i x^{2^{k+i}+2^i}$	$n = 2k$, $\gcd(s,k) = 1$, s, k odd, $\beta \notin \mathbb{F}_{2^k}$, $\gamma_i \in \mathbb{F}_{2^k}$, α not a cube

CCZ-inequivalent APN functions (I)

Table 4. CCZ-inequivalent APN functions on $\mathbb{F}_{2^n}$ from known APN families ($6 \leq n \leq 11$) [Budaghyan, Helleseth, Li, Sun 2017]

n	N°	Functions	Families from Tables 1-2	Relation to [*]
6	6.1	x^3	Gold	Table 5: $N^\circ 1.1$
	6.2	$x^6 + x^9 + a^7 x^{48}$	$N^\circ 3$	5: $N^\circ 1.2$
	6.3	$ax^3 + a^4 x^{24} + x^{17}$	$N^\circ 8$-10	5: $N^\circ 2.3$
7	7.1	x^3	Gold	Table 7 : $N^\circ 1.1$
	7.2	x^5	Gold	7 : $N^\circ 3.1$
	7.3	x^9	Gold	7 : $N^\circ 4.1$
	7.4	x^{13}	Kasami	7 : $N^\circ 5.1$
	7.5	x^{57}	Kasami	7 : $N^\circ 6.1$
	7.6	x^{63}	Inverse	7 : $N^\circ 7.1$
	7.7	$x^3 + \mathrm{tr}_1^7(x^9)$	$N^\circ 5$	7 : $N^\circ 1.2$
8	8.1	x^3	Gold	Table 9 : $N^\circ 1.1$
	8.2	x^9	Gold	9 : $N^\circ 1.2$
	8.3	x^{57}	Kasami	9 : $N^\circ 7.1$
	8.4	$x^3 + x^{17} + a^{48} x^{18} + a^3 x^{33} + ax^{34} + x^{48}$	$N^\circ 4$	9 : $N^\circ 2.1$
	8.5	$x^3 + \mathrm{tr}_1^8(x^9)$	$N^\circ 5$	9 : $N^\circ 1.3$
	8.6	$x^3 + a^{-1}\mathrm{tr}_1^8(a^3 x^9)$	$N^\circ 5$	9 : $N^\circ 1.5$

a: primitive root of $\mathbb{F}_{2^n}$;
[*]: Edel, Y., Pott, A.:"A New Almost Perfect Nonlinear Function Which Is Not Quadratic".

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CCZ-inequivalent APN functions (II)

Table 4. Continued

n	N°	Functions	Families from Tables 1-2
9	9.1	x^3	Gold
	9.2	x^5	Gold
	9.3	x^{17}	Gold
	9.4	x^{13}	Kasami
	9.5	x^{241}	Kasami
	9.6	x^{19}	Welch
	9.7	x^{255}	Inverse
	9.8	$x^3 + \mathrm{tr}_1^9(x^9)$	$N^\circ 5$
	9.9	$x^3 + \mathrm{tr}_3^9(x^9 + x^{18})$	$N^\circ 6$
	9.10	$x^3 + \mathrm{tr}_3^9(x^{18} + x^{36})$	$N^\circ 7$
10	10.1	x^3	Gold
	10.2	x^9	Gold
	10.3	x^{57}	Kasami
	10.4	x^{339}	Dobbertin
	10.5	$x^6 + x^{33} + a^{31}x^{192}$	$N^\circ 3$
	10.6	$x^{72} + x^{33} + a^{31}x^{258}$	$N^\circ 3$
	10.7	$x^3 + \mathrm{tr}_1^{10}(x^9)$	$N^\circ 5$
	10.8	$x^3 + a^{-1}\mathrm{tr}_1^{10}(a^3x^9)$	$N^\circ 5$

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CCZ-inequivalent APN functions (III)

Table 4. Continued

n	N°	Functions	Families from Tables 1-2
11	11.1	x^3	Gold
	11.2	x^5	Gold
	11.3	x^9	Gold
	11.4	x^{17}	Gold
	11.5	x^{33}	Gold
	11.6	x^{13}	Kasami
	11.7	x^{57}	Kasami
	11.8	x^{241}	Kasami
	11.9	x^{993}	Kasami
	11.10	x^{35}	Welch
	11.11	x^{287}	Niho
	11.12	x^{1023}	Inverse
	11.13	$x^3 + \mathrm{tr}_1^{11}(x^9)$	$N^\circ 5$

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Classification of APN Functions for Small n

Completed

- $n \leqslant 5$: only power APN functions [Brinkmann,Leander 2009]
- $n = 6$, quadratic APN functions (13 classes).

Open

- Many unclassified quadratic APN polynomials for $6 < n \leqslant 12$ [Dillon et all 2006, Edel and Pott 2009; Yu et all 2013].
- One known example of quadratic APN function (with n = 6) with non-Gold like nonlinearity [Dillon et al 2006].
- One known example of APN polynomial CCZ-ineq. to quadratics and to power functions (n=6) [Leander et al 2008; Edel and Pott 2009].
- One known example of APN permutations for n even (with n = 6, CCZ-eq. to quadratics!) [Dillon et al 2009].

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Trinomials

Quadratic Trinomial Functions

$F = x^{2^{i_1}+1} + x^{2^{j_2}(2^{i_2}+1)} + x^{2^{j_3}(2^{i_3}+1)}$, on $\mathbb{F}_{2^n}$ to itself. n is from 6 to 11.

Conditions for efficiency:

- if n is even: $i_1 \leqslant i_2, i_3 \leqslant \frac{n}{2}$;
  if n is odd: $i_1 \leqslant i_2, i_3 \leqslant \frac{n-1}{2}$;
- $0 \leqslant j_2, j_3 \leqslant n-1$;
- $2^{i_1} + 1 < 2^{j_2}(2^{i_2} + 1) \mod (2^n - 1) < 2^{j_3}(2^{i_3} + 1) \mod (2^n - 1)$;

Quadrinomials, Pentanomials, Hexanomials

With similar conditions as trinomial, on $\mathbb{F}_{2^n}$ to itself, $6 \leqslant n \leqslant 11$:

Quadratic Quadrinomials

$$F = x^{2^{i_1}+1} + x^{2^{j_2}(2^{i_2}+1)} + x^{2^{j_3}(2^{i_3}+1)} + x^{2^{j_4}(2^{i_4}+1)};$$

Quadratic Pentanomials

$$F = x^{2^{i_1}+1} + x^{2^{j_2}(2^{i_2}+1)} + x^{2^{j_3}(2^{i_3}+1)} + x^{2^{j_4}(2^{i_4}+1)} + x^{2^{j_5}(2^{i_5}+1)};$$

Quadratic Hexanomials

$$F = x^{2^{i_1}+1} + x^{2^{j_2}(2^{i_2}+1)} + x^{2^{j_3}(2^{i_3}+1)} + x^{2^{j_4}(2^{i_4}+1)} + x^{2^{j_5}(2^{i_5}+1)} + x^{2^{j_6}(2^{i_6}+1)}.$$

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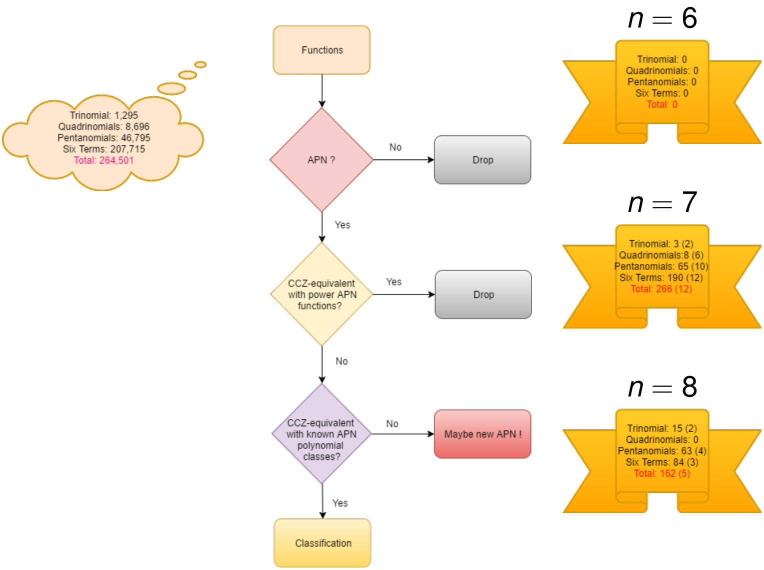
Procedures, $n = 6, 7, 8$



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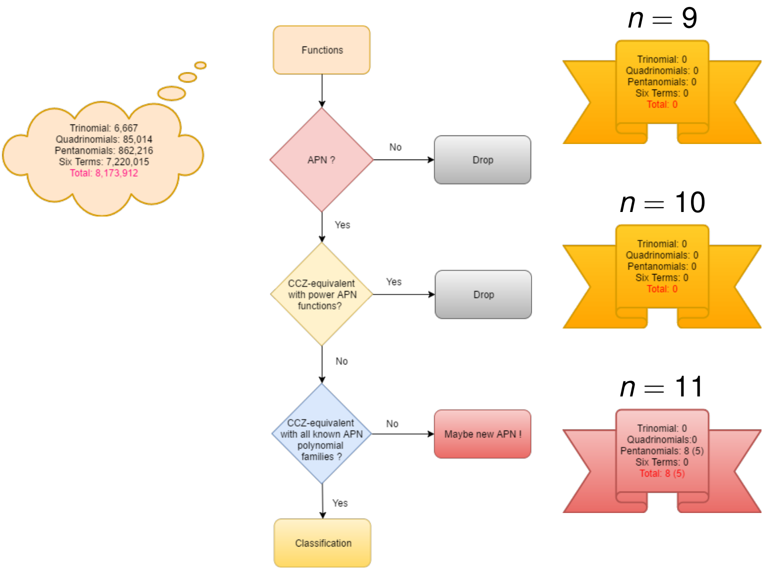
Procedures, $n = 9, 10, 11$



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   - Cryptosystems and S-boxes
   - Vectorial Boolean Functions and Attacks
   - Nonliearity
   - Differential Uniformity
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   - Three Equivalences
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Result (I)

Table 5. A number of APN polynomials (up to CCZ-equivalence) which are not CCZ-equivalent to power functions.

n	Number of Terms	Number of Polynomials
6	3 – 6	–
7	3	2
	4	6
	5	10
	6	12
8	3	2
	4	–
	5	4
	6	3
9, 10	3 – 6	–
11	3	–
	4	–
	5	5
	6	–

Result (II)

Table 6. A number of APN polynomials (up to CCZ-equivalence) which are not CCZ-equivalent to APN polynomials in fewer terms with coefficients in $\mathbb{F}_2$.

n	Number of Terms	Number of Polynomials
6	$3-6$	–
7	3	2
	4	5
	5	4
	6	1
8	3	2
	4	–
	5	2
	6	1
9, 10	$3-6$	–
11	3	–
	4	–
	5	5
	6	–

5 New APN Polynomials on $\mathbb{F}_{2^{11}}$

Table 7. New 5 APN polynomials (up to CCZ-equivalence) on $\mathbb{F}_{2^{11}}$.

Classes NO.	Polynomials
1	$x^{12} + x^{10} + x^9 + x^5 + x^3$ $x^{1536} + x^{1026} + x^{514} + x^{513} + x^3$
2	$x^{258} + x^{257} + x^{18} + x^{17} + x^3$
3	$x^{96} + x^{66} + x^{34} + x^{33} + x^3$ $x^{192} + x^{130} + x^{129} + x^{65} + x^3$
4	$x^{80} + x^{68} + x^{65} + x^{17} + x^5$ $x^{640} + x^{516} + x^{132} + x^{129} + x^5$
5	$x^{260} + x^{257} + x^{36} + x^{33} + x^5$

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Thank you!